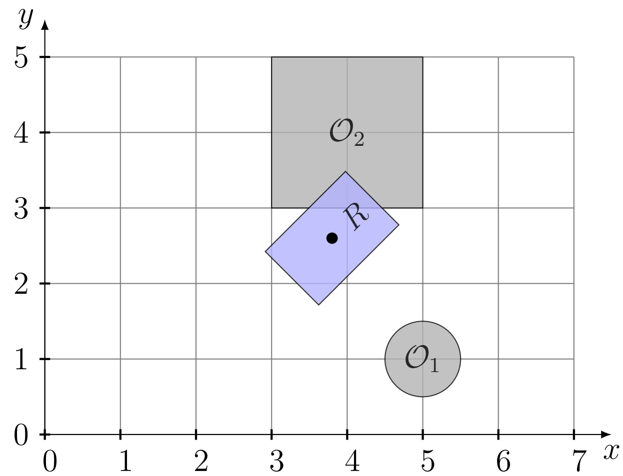
Motion Planning — Exercise 12

Wolfgang Hönig and Andreas Orthey, TU Berlin SS 2024

1. In the lecture we discussed the differential flatness for a unicycle. We will now extend this to the case of car dynamics as defined in Lecture 1 (slide 35).
   (a) Prove that the kinodynamic car is differentially flat using the flat outputs $\mathbf{z}(t) = (x, y)$ by defining g_q and g_u.
   (b) Explain how you can use this result for motion planning with polynomial spline optimization. What is the minimum order n needed for your polynomials?
   (c) Explain how you can use a) and b) for sampling-based planning with RRT* for your kinodynamic car.
   (d) Explain how you can use a) and b) for search-based motion planning.
2. Consider the following special dynamics and decide if differential flatness is i) applicable in all cases, ii) applicable in most cases (define which ones), or iii) not applicable. Explain your decision, and if differential flatness is applicable state the achieved dimensionality reduction of the motion planning problem.
   (a) A basic model of an airplane cruising at a fixed altitude which can be modeled as a unicycle, where the action space is constrained to be strictly positive, e.g., $s \in [100, 250]\,\mathrm{km/h}$.
   (b) A 2D double integrator that can accelerate with up to $1\,\mathrm{m/s^2}$ in x, but only $0.5\,\mathrm{m/s^2}$ in y.
   (c) A car with a partially broken steering wheel, where $\phi \in [-\pi/6, \pi/12]$.

3. Consider the following case with two static obstacles $\mathcal{O}_1$ and $\mathcal{O}_2$ and robot R, where the initial guess places the robot on top of one obstacle.



(a) Compute the linearized signed distance function $sd_1(\mathbf{q})$ with respect to $\mathcal{O}_1$. Use the diagram to (roughly) estimate the points of contacts. Assume that the robot has car dynamics (i.e., $\mathbf{q} = (x, y, \theta)$) with the center of mass being the reference point.

(b) Repeat the same for the signed distance function $sd_2(\mathbf{q})$.

(c) Explain how SCP would guide the updated configuration $\mathbf{q}'$ based on just the constraints imposed by $sd_1(\mathbf{q})$ and $sd_2(\mathbf{q})$.

(d) What changes if the robot model is a single integrator (i.e., $\mathbf{q} = (x, y)$)?