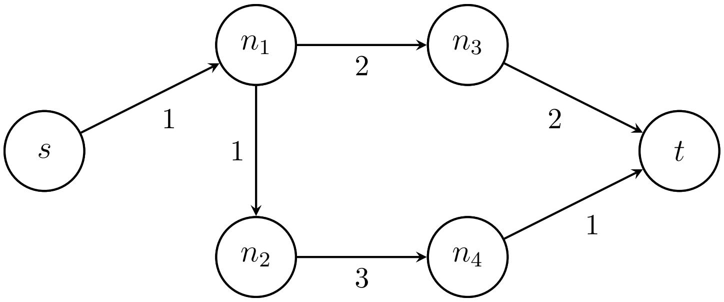
MOTION PLANNING — EXERCISE 5

Wolfgang Hönig and Andreas Orthey, TU Berlin SS 2024

Non-Programming

1. Consider the following graph, where numbers on the edges represent the cost to traverse an edge.



In all cases, the goal is to compute the minimum-cost path from s to t. Let the heuristic be $h(s) = 4, h(n_1) = 4, h(n_2) = 1, h(n_3) = 2, h(n_4) = 0.5, h(t) = 0$.

   (a) Document the execution of weighted A* with $\epsilon = 2.0$ by writing down the OPEN list, $g(\cdot)$, and $f(\cdot)$ in each iteration.

   (b) Document the execution of weighted A* with $\epsilon = 1.1$ by writing down the OPEN list, $g(\cdot)$, and $f(\cdot)$ in each iteration.

   (c) Document the execution of ARA* with $\epsilon = \langle 2.0, 1.1 \rangle$ by writing down the OPEN list, $g(\cdot)$, $g_e(\cdot)$, and $f(\cdot)$ in each iteration. How does the computational effort compare to the two independent wA* executions?

2. This question concerns the number of motion primitives that are needed in different settings.

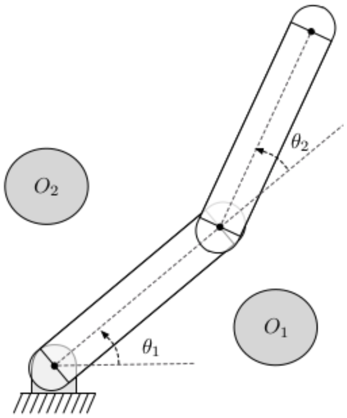
   (a) Consider the car from exercise 1 with state (x, y, θ). What is an upper bound of motion primitives needed if you discretize the position by 8 m and the orientation by 90 deg?
   Hint: A reasonable upper bound can ignore if a desired motion is indeed feasible.
   Hint: Note that the car dynamics are translation-invariant.

   (b) Consider the same case as in a), but with an orientation discretization of 5 deg.

   (c) Now consider a second-order car with state $(x, y, v, \theta, \omega)$, that is position, speed, orientation, angular velocity. What is an upper bound of motion primitives needed if you discretize the position by 8 m, the orientation by 90 deg, and allow $s \in \{-0.5, 1.0, 2.0\}$ m/s and $\omega \in \{-0.5, 0, 0.5\}$ rad/s?

(d) Consider the same case as in c), but with $s \in \{-0.5, -0.25, 0.25, 0.5, 1.0, 1.5, 2.0\}$ m/s.

3. How can search-based planning be applied to a 2-link robotic manipulator?



What is the main challenge compared to the car examples?

4. Mathematically define a non-trivial heuristic for minimum-time movement of a second-order kinodynamic system, i.e., a robot that has bounds on the acceleration and velocity magnitudes.

5. Consider a car with dynamics defined in lecture 1. Assume you have 12 motion primitives: the yaw-angle θ is discretized in 4 directions $(0, \pi/2, \pi, 3/2\pi)$ and for each of the 4 directions there are three primitives: a left turn, a right turn, and a motion forward using a square grid size of 8 m.

   (a) Provide pseudo code on how the 12 primitives can be generated. You may use a `integrate` helper function. A turn can be achieved in 6.29 s by turning the steering wheel $\pi/8.75$ radians and moving with 2 m/s.

   (b) Draw the implicitly constructed search graph when applying lattice-state A* on the following environment, assuming $h(\cdot)$ uses the minimum-time first-order approximation defined in the lecture and $s_{max} = 2$ m/s. The start state is $(8, 16, 0)$ and the goal state is $(40, 16, \frac{\pi}{2})$.