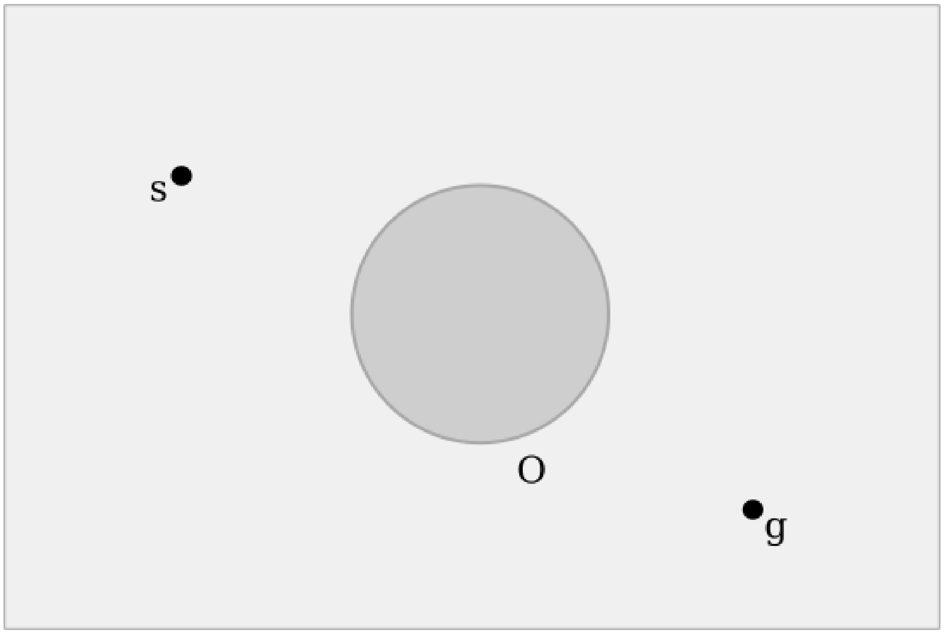
Motion Planning — Exercise 3

Wolfgang Hönig and Andreas Orthey, TU Berlin SS 2024

Non-Programming

1. Provide the dynamics $\mathbf{f}$, configuration space, degrees of freedom, and action space for the following systems.
   Hint: You can use, e.g., LaValle's Planning Algorithms book to find common definitions.
   (a) Unicycle (1^{st} order)
   (b) Unicycle (2^{nd} order)
   (c) Double integrator in 2D
   (d) Car (2^{nd} order)
   (e) Car with trailer

2. **The following question's parts (a) to (c) are identical to exercise sheet 1, question 5 (which has not been discussed, yet). Parts (d) and (e) are new.**
   The goal of this question is to clarify the relationship between geometric and kinodynamic motion planning.
   (a) A 1-dimensional single-integrator has the dynamics $\dot{x} = f(x, u) = u$; in other words, the velocity can be controlled directly. Provide the configuration space, degrees of freedom, action space, and configuration map for a point robot in 2D with single-integrator dynamics.
   (b) Apply the example from a) to the definition of kinodynamic motion planning. State the minimal set of assumptions that make the problem equivalent to the definition of geometric motion planning.
   (c) Is the inverse of b) possible, i.e., can you construct a geometric motion planning instance that solves any given kinodynamic motion planning instance? If yes, provide a description of your construction. Otherwise, argue that such a construction cannot be made.
   Hint: Consider changing the configuration and configuration map.
   (d) For your solution in b), provide a concrete example on how a kinodynamic motion planner can be used to solve the following geometric problem in 2D. Here, the robot is circular with radius r_1 and needs to avoid a single obstacle with radius r_2.



(e) For your solution in c), consider the case where a motion plan should be computed for a kinodynamic car (dynamics as defined in lecture 1) using a geometric planner. Use the same example as in (d) (i.e., the robot and single obstacle are both circular). If that is possible, provide a concrete description for all inputs for the motion planner. If no such conversion is possible, clearly explain the reasons using that example.