Motion Planning — Exercise 1

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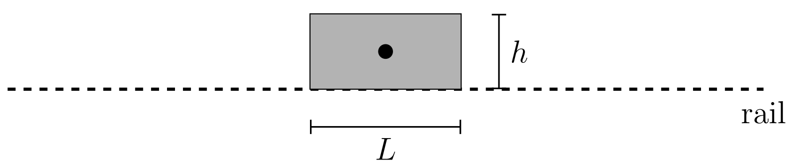
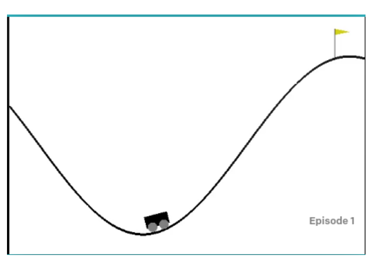
Non-Programming

1. Provide the configuration space, degrees of freedom, and action space for the following systems.
   (a) Differential-drive (e.g., vacuum cleaning robot).
   (b) Train.
   (c) Multirotor.
2. Provide the configuration map for a train of length L and height h.



3. The mountain car is a frequently used example for reinforcement learning. Here, a car is tasked with driving up a hill, but does not have sufficient acceleration to reach the goal directly. Instead, a valid solution requires the car to first go backwards to gain some momentum.



   (a) Provide the configuration space, degrees of freedom, and action space for the mountain car.
   (b) Define the configuration map.
   (c) Frame the mountain car problem as a motion planning instance. To this end, explain if this is a geometric or kinodynamic problem.
4. Provide mathematical definitions for

(a) The time objective $J(T, \mathbf{u}(t), \mathbf{q}(t))$ in kinodynamic planning.

(b) The path length objective $J(\mathbf{q}(p))$ in geometric planning. Hint: You may introduce helper functions to extract parts of the configuration.

5. The goal of this question is to clarify the relationship between geometric and kinodynamic motion planning.

   (a) A 1-dimensional single-integrator has the dynamics $\dot{x} = f(x, u) = u$; in other words, the velocity can be controlled directly. Provide the configuration space, degrees of freedom, action space, and configuration map for a point robot in 2D with single-integrator dynamics.

   (b) Apply the example from a) to the definition of kinodynamic motion planning. State the minimal set of assumptions that make the problem equivalent to the definition of geometric motion planning.

   (c) Is the inverse of b) possible, i.e., can you construct a geometric motion planning instance that solves any given kinodynamic motion planning instance? If yes, provide a description of your construction. Otherwise, argue that such a construction cannot be made.
   Hint: Consider changing the configuration and configuration map.